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Corrigendum

Zeno dynamics in quantum statistical mechanics
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(I) On page 1137, the argument in line 4 from below starting with 'Thus we obtain ...', and including the following two formulae, must be replaced by the following:

Multiplying out and using repeatedly $\|AB\| \leqslant \|A\|\|B\|$, we estimate this expression from above by

$$\|[EU(t/n)E]\|^{n-k} \cdot \|EU(t(m-l)/(nm))E^{\perp}\| \cdot \|E^{\perp}U(t/(nm))E\| \\ \cdot \|[EU(t/(nm))E]\|^{l-1}\|[EU(t/(nm))E]\|^{m(k-1)}.$$

Observing that all terms containing only the projection E have operator norm $\leqslant 1$ and can thus be omitted in the estimation of $\|(F_n(t) - F_{nm}(t))\|$, we arrive at

$$\|\|(F_n(t) - F_{nm}(t)) \leqslant \sum_{k=1}^{n}\sum_{l=1}^{m-1} \|EU(t(m-l)/(nm))E^{\perp}\|\|E^{\perp}U(t/(nm))E\|.$$

(II) Equation (2) on page 1138 must be replaced by the estimate

$$\|E^{\perp}U(\zeta)EA\Omega\| \leqslant C \cdot \|A\Omega\| \cdot |\zeta|. \tag{2}$$

(III) On page 1139, insert the following paragraph before the last paragraph of section 2:

The AZC, when restricted to the real axis, is equivalent to saying that the function $E^{\perp}U(t)E$ is uniformly Lipschitz continuous at the point $t = 0$. Not surprisingly, Lipschitz continuity is well known as a salient condition for the existence of solutions to (nonlinear) evolution equations.

(IV) On page 1141, replace the last sentence of example 2 (lines 16–20) as follows:

Choose a faithful normal state for the global algebra $\mathcal{A}$, and let U be the unitary generating the global dynamics τ in its GNS representation. Under the assumptions above we obtain that the Zeno limit

$$W_{\varphi_\Lambda}(t) = \lim_{n\to\infty}\left[E_{\varphi_\Lambda}U(t/n)E_{\varphi_\Lambda}\right]^n$$

exists, where $E_{\varphi_\Lambda} := \lim_{\Lambda'\to\infty} E_{\varphi_\Lambda} = \mathbf{1}_{\Lambda^c} \otimes P_{\Phi_\Lambda}$. It defines an automorphism group τ^E of $\mathcal{A}$. Furthermore, this automorphism group is the uniform limit of the local automorphism groups $\tau^{E_{\varphi_\Lambda;\Lambda'}}$ defined by $W_{\varphi_\Lambda;\Lambda'}$.

(V) The calculation in the proof of proposition 4.1 is too terse and not correct as it stands. On page 1143, line 11 from below starting with 'Omitting terms ...', and the following formula, have to be replaced by the following:

The norm of the vector under the sum is, using (4),

$$\|[EU(t/n)E]^{n-i}(EU(t/n)E - EU_E(t/n)E)\Psi_E(t(i-1)/n)\|.$$

Using commutativity of U_E with E, and the invariance of $\Psi_E(\sigma)$ under E, we have $EU_E(t/n)E\Psi_E(t(i-1)/n) = U_E(t/n)\Psi_E(t(i-1)/n)$, and use this to rewrite the above expression as

$$= \|[EU(t/n)E]^{n-i}(EU(t/n)E - U_E(t/n))\Psi_E(t(i-1)/n)\|.$$

Now, with $\|[EU(t/n)E]^{n-i}\| \leqslant 1$ and $\|AB\Psi\| \leqslant \|A\|\|B\Psi\|$, this is bounded by

$$\leqslant \|(EU(t/n)E - U_E(t/n))\Psi_E(t(i-1)/n)\|.$$

Putting this together, we obtain the estimate

$$\|F_n(t)\Psi_E - U_E(t)\Psi_E\| \leqslant \sum_{i=1}^{n} \|(EU(t/n)E - U_E(t/n))\Psi_E(t(i-1)/n)\|.$$

From there, proceed as in the original text with 'We can now apply ...'.

(VI) On page 1145, replace corollary 5.2 as follows:

Corollary 5.2. *Let $\beta > 0$. Let $\Lambda_\alpha \to \infty$ be such that the local dynamics converges uniformly to the global dynamics τ, and the net of local Gibbs states ω_{Λ_α} has a thermodynamic limit point ω. Let U be the unitary group representing τ in the GNS representation of ω. If a sequence of projections $E_{\Lambda_\alpha} \in \mathcal{A}(\Lambda_\alpha)$ converges in norm to a projection E in $\mathcal{A}$ such that (U, E) is regular and satisfies (AZC), then $\omega_E(A_E) := \lim_\alpha \omega^G_{E_{\Lambda_\alpha}}(A_E)$ defines a (τ^E, β)-KMS state on $\mathcal{A}_E$.*

Proof. The local Gibbs states $\omega_{E_{\Lambda_\alpha}}$ are the unique β-KMS states on the finite-dimensional algebras $\mathcal{A}_{\Lambda_\alpha}$ for the reduced dynamics $\widehat{\tau}^{E_{\Lambda_\alpha}}$. If $\{E_{\Lambda_\alpha}\}$ converges uniformly, these local Gibbs states possess ω_E as a weak-* limit, which is a KMS state on $\mathcal{A}_E$ for the reduced dynamics $\widehat{\tau}^E$ associated with τ. Then, by corollary 5.1, ω_E is also a (τ^E, β)-KMS state. □

(VII) Replace the part of example 5 on page 1146 between 'By using ...' in line 10 up to the sentence before 'A similar result ...' in line 20 by the following:

As in example 4, the local Zeno Hamiltonians decompose into two commuting, nontrivial parts over the subchains $[-m, -1]$ and $[1, n]$ which are averaged with respect to ρ_0, and a scalar part

$$E_{\rho_0} H E_{\rho_0} = H^{\rho_0}_{-m} + H^{\rho_0}_0 + H^{\rho_0}_{+n},$$

where E_{ρ_0} is restricted to $\mathcal{H}_{[-m,n]}$ in the natural way. Explicitly we obtain $H^{\rho_0}_0 = h\rho_0(a_0^* a_0)$, and

$$H^{\rho_0}_{+n} = \frac{J}{2}(\overline{\rho_0(a_0)}a_1 + \rho_0(a_0)a_1^*) + H_{[1,m]},$$

and likewise for $H^{\rho_0}_{-m}$. Straightforwardly, we obtain Gibbs states over the left and right subchains:

$$\omega^+_{\rho_0,\beta}(A_+) := \lim_{n\to\infty} \frac{\mathrm{Tr}_{\mathcal{H}_{[1,n]}}\left(\mathrm{e}^{-\beta H^{\rho_0}_{+n}} A_{+n}\right)}{\mathrm{Tr}_{\mathcal{H}_{[1,n]}}\left(\mathrm{e}^{-\beta H^{\rho_0}_{+n}}\right)},$$

where $\{A_{+n} \in \mathcal{A}_{[1,n]}\}$ converges to A_+ in $\mathcal{A}_{[1,\infty]}$, which is the weak closure of the union of the local algebras $\mathcal{A}_{[1,n]}$.